Mech 1.

a) Conservation of Mechanical Energy:

$$\Delta U = KE$$

$$m_1 gL = \frac{1}{2} m_1 v_1^{\;2}$$

$$v_1 = \sqrt{2gL}$$

b) Newton's Second Law:

$$\sum F_y = ma$$

$$T - m_1 g = m_1 \frac{v_1^{\;2}}{L}$$

$$T - m_1 g = m_1 \frac{2gL}{L}$$

$$T = 3mg$$

c) Conservation of Linear Momentum:

$$P_1 = P_2$$

$$m_1 v_1 = (m_1 + m_2) v_2$$

$$m_1 \sqrt{2gL} = (m_1 + m_2) v_2$$

$$v_2 = \frac{m_1}{m_1 + m_2} \sqrt{2gL}$$

d)

$$KE_1 = \frac{1}{2} m_1 v_1^{\;2} = m_1 gL$$

$$KE_2 = \frac{1}{2}(m_1 + m_2) v_2^{\;2} = \frac{m_1 + m_2}{2} \frac{m_1^{\;2}\, 2gL}{(m_1 + m_2)^2} = \frac{m_1^{\;2} gL}{m_1 + m_2}$$

$$\frac{KE_1}{KE_2} = \frac{m_1 + m_2}{m_1}$$

e)

$$\Delta y = v_0 - \frac{1}{2} gt^2$$

$$-L = -\frac{1}{2} gt^2$$

$$t = \sqrt{\frac{2L}{g}}$$

$$\Delta x = L + v_2 t = L + \frac{m_1}{m_1 + m_2} \sqrt{2gL} \sqrt{\frac{2L}{g}} = L + \frac{2m_1 L}{m_1 + m_2}$$

Mech 2.

a)

$$\Delta y = v_0 - \frac{1}{2}gt^2$$

$$-D = -\frac{1}{2}gt^2$$

$$a = \frac{2D}{t^2}$$

b) i. D and t^2. Slope of graph is a/2

ii. Plot D with respect to t^2, draw the best-fit line.

iii. 2.08 m/s^2

c)

$$\sum F_y = ma$$

$$T - mg = -ma$$

$$T = mg - ma$$

$$\tau = r \times F = I\alpha$$

$$RT = I\frac{a}{R}$$

$$I = \frac{R^2 T}{a} = \frac{(mg - ma)R^2}{a}$$

d) Friction between string and pulley.

Mech 3.

a) Parallel Axis Theorem:

$$I_d = I_{CM} + md^2$$

$$I = \frac{ML^2}{12} + M(\frac{L}{2} - \frac{L}{3})^2 = \frac{ML^2}{9}$$

b)

$$\tau = r \times F = \frac{L}{6}Mg\sin(90 - \theta)$$

$$W = \int \tau \cdot d\theta = \frac{L}{6}Mg\int_0^{90}\sin(90 - \theta) \cdot d\theta = \frac{MgL}{6}$$

$$W = KE$$

$$\frac{MgL}{6} = \frac{1}{2}I\omega^2$$

$$\frac{MgL}{6} = \frac{1}{2}\frac{ML^2}{9}\omega^2$$

$$\omega = \sqrt{\frac{3g}{L}}$$

$$v = r\omega = \frac{2L}{3}\sqrt{\frac{3g}{L}} = \frac{2}{3}\sqrt{3gL}$$

c) If ϕ is small:

$$\tau = r \times F = \frac{L}{6} Mg \sin(90 - \theta) = \frac{MgL\sin(\phi)}{6} = I\alpha$$

$$\alpha = \frac{MgL\sin(\phi)}{6I} = \frac{MgL\sin(\phi)}{6}\frac{9}{ML^2} = \frac{3g}{2L}\sin(\phi) \approx \frac{3g}{2L}\phi$$

$$\frac{d^2\phi}{dt^2} = \frac{3g}{2L}\phi$$

$$T = 2\pi\sqrt{\frac{2L}{3g}}$$